Icelandic Research Fund

External Peer Review

RANNIS reference:	141761-051
Principal investigator:	Henning Arnór Úlfarsson
Organisation:	Reykjavik University-School of Computer science
Project title:	Finding structure in sets of permutations
Expert Panel:	Engineering, technical sciences and sciences
Fund:	Project grant
Program Officer:	Sigurður Björnsson
Contact information:	sigurdur@rannis.is or +354 515 5801

Part 1 - In depth

Originality of the project **Grade: 5 of 5**

One of the main goals of this project is the development of the software package Struct, which will automatically deduce "rules" for the structure of an input set of permutations. This package will be significantly different from the existing enumeration algorithms (primarily the insertion encoding, the substitution decomposition, geometric grid classes, and enumeration schemes). In particular, no such algorithm exists today, and so Struct will be very novel, and potentially have a large impact on the field. I am particularly interested in the "mutation rules" intended to be developed for Struct (Extension C), and the implementation of parallelism (Extension D) will make Struct much more applicable. Even if obtaining unique structural rules for sets of permutations (a requirement for Extension A) proves difficult, the proposers sensibly point out that it would still lead to quite beneficial insights, and to upper and lower bounds.

In addition, the proposers intend to extend the BiSC package to handle "decorated patterns", with a package called BiSCd (Extension B). I agree with the proposers' belief that BiSCd should be able to discover both old and new theorems, which could be quite interesting.

Finally, I can't stress enough that despite the growing power of computers, "automated combinatorics" (i.e., proving combinatorial theorems by computer) has remained an underdeveloped field. The proposers' are some of the very few combinatorialists exploring this much-needed area of research.

State of the art **Grade: 5 of 5**

This proposal has the potential to have wide-ranging impact on permutation patterns, which is itself a very diverse field. Thus to describe the "state of the art" of the field, I will stick to automatic or systematic methods to find and/or describe the structure of a given set of permutations.

As the proposal does a good job of describing, there are four major techniques for this: the insertion encoding, the substitution decomposition, geometric grid classes, and enumeration schemes. The first three of these methods require that the input set of permutations be specified by avoiding classical patterns. The fourth has been extended to handle some sets which are specified by slightly more general patterns, but these patterns have nowhere near the expressive power of mesh patterns. Thus the Struct package (and the associated BiSCd

package) described in the proposal will represent a true advance on the state of the art.

The methodology seems entirely appropriate for research in pure mathematics, and the extensions to Struct seem very sensible.

Principal investigator and project management **Grade: 5 of 5**

The PI, Ulfarsson, is uniquely qualified for the project under consideration, as is demonstrated by several factors. First, the PI has already developed and implemented the BiSC algorithm, upon which this project is based. Second, the PI has shown to be quite good at giving characterizations of various sets of permutations (such as the locally complete intersection Schubert varieties and the West-3-stack-sortable permutations).

I am also greatly encouraged by the participation of the first Co-PI, Albert. Albert has extensive experience with almost all aspects of the combinatorics of permutation patterns, and his importance to the field is witnessed by his chairmanship of the steering committee of the annual Permutation Patterns conference. Albert has extensive development experience himself, having written the software package PermLab, which is used extensively in the study of permutation patterns.

The other Co-PI, Claesson, is also a very accomplished combinatorialist, specializing in permutation patterns. He is, more than anyone else, responsible for the development of the language of mesh patterns.

Deliverables and impact **Grade: 5 of 5**

Two of the main deliverables will be the software packages Struct and BiSCd. These two packages could prove very useful for other researchers in the field. Indeed, the PI's package BiSC has already proved very useful for researchers in permutation patterns. Also, the proposers intend to develop parallel versions of the packages, which could help to greatly reduce their run-times, making running the algorithms much more practical than it otherwise would be.

In addition to those software packages, the proposers promise the "standard" deliverables in a pur mathematics project, talks at international conferences and papers published in reputable peer-reviewed journals. The proposers all have a strong background of talks and papers, so I have no doubt that this project will lead to many such deliverables.

Part 2 - Summary

Overall strengths

The combinatorics of permutation patterns is a lively and fast-growing area of combinatorics. Two of the biggest questions in this area are:

1. Given a set of permutations, how can they be characterized by a set of "forbidden patterns" of some type?

2. Given a set of forbidden patterns, how can the permutations avoiding them be characterized, and can they be enumerated?

As demonstrated already in the mid-90s, for certain natural sets of permutations, "classical" permutation patterns do not suffice as a characterization. The first example demonstrated by this were the West-2-stack-sortable permutations, though many even more intriguing examples have emerged since, such as the characterization of factorial Schubert varieties and simsun permutations. The PI has been part of a group developing the theory "mesh patterns", which have shown great success in characterizing such sets of permutations.

The potential insight gained from the proposed research is great. For example, the proposal highlights the challenge of providing new bounds on 1324-avoiding permutations. If the software package Struct is able to do this, that would be a large advance in the field.

It should also be noted that the proposers are the only researchers investigating this interesting direction.

Overall weaknesses

The only possible weakness I can see in this proposal is that the project might lead to interesting new questions that enlarge the project scope causing it to be too large to fully complete in the planned time scale. However, this is always a danger/opportunity when conducting novel and ambitious research.

Overall score

Grade: 20 of 20